5. *Fixation* or removal of remaining halide.*

In conclusion, the authors desire to express their great thanks to Sir William Ramsay, K.C.B., F.R.S., for his constant advice and interest in the investigation.

The Relation between Breaking Stress and Extension in Tensile Tests of Steel.

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A large number of the tensile tests of steel are now made with test-pieces, which are only a few diameters long (fig. 1).

Fig. 1.

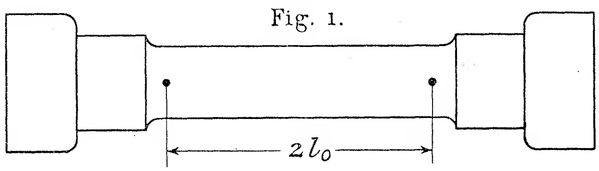
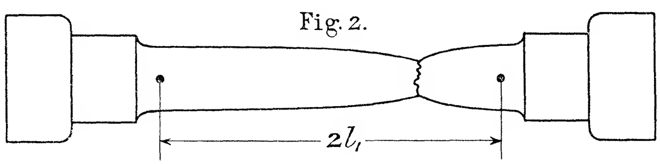


Fig. 2.



When such a test-piece is broken by tension, it has a profile, as shown in fig. 2. The usual records, made when the tests are carried out, include, among other things, "breaking stress" and "extension per cent."

"Breaking stress" here means the maximum tension applied divided by the original area of the test-piece; and extension per cent. is taken as the percentage increase due to the strain, in the distance between two marks, one at either end of the test-piece, whose unstrained distance is known. The use of the term "breaking stress" in the above sense is convenient, from an engineer's point of view, as showing what force a bar, etc., of given sectional area will stand before giving way. The true breaking stress of a material, however, is the actual intensity of the stress at

* 'Phot. Journ.' (Trans. Roy. Phot. Soc.), 1906, vol. 46, p. 235: "On the Theory of Fixation."

the broken surface, and is, of course, greater than the nominal breaking stress, because of the reduced area of the broken surface. To avoid confusion, I will call the true breaking stress the "intrinsic strength" of the material.

An examination of a very large number of observations made with the short test-pieces shows that, if the nominal breaking stress (as defined above) is expressed in tons per square inch, the arithmetical sum, breaking stress + elongation per cent., remains constant, and equal to about 67 or 68 for all mild steels, which, at the beginning of the test, are free from internal mechanical strain, no matter what has been the heat treatment of hardening and annealing to which they have been subjected.

The object of this note is to examine the reason for this: for, since breaking stress has the dimensions of a force $\div$ an area, and extension per cent. is a pure number, it seems at first sight that no physical quantity could be represented by their sum.

To determine the relation between breaking stress (B), elongation per cent. (E), and the intrinsic strength of the material, the form assumed by the test-piece when extended must be known.

The experimental fact to be explained is $B+100E_2$ = constant = 67 or 68, if B is expressed in tons per square inch.

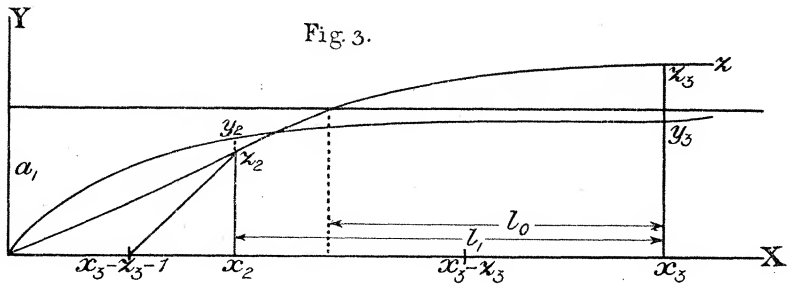
Any relation which makes $dB/dE = -1$ ensures the constancy of the sum of $B+E$, and a variety of relations might be assumed which will do this approximately; but the particular relation to be sought for is that which not only makes $dB/dE = -1$, but also makes the diameter calculated from E correspond to the measured diameter, not only at the break, but along the whole length of the test-piece.

When the extension is a small percentage of the whole length, the contraction of the diameter is nearly uniform over the whole length. As the extension proceeds, the local contraction (fig. 2) appears, and breakage ultimately occurs at the narrowest part of the neck.

Measurements taken from a large number of test-pieces show that the extensions can be represented as being due to (1) a general and uniform contraction of the diameter of the test-piece, and (2), in addition to this, a contraction of diameter, which at any given cross-section is a negative exponential function of the distance of that cross-section from the cross-section where the break occurs, the axis of the exponential curve being the generating line of the cylinder, to which the distant sections of the test-piece (had it been long) would have been reduced by (1).

The extensions corresponding to (1) and (2) will be considered separately.

Fig. 3.



Let $2l_0$ = original length of test-piece,

$2l_1$ = extended „ „

a_0 = original diameter „

a_1 = contracted diameter „

$E_1 = \dfrac{l_1 - l_0}{l_1}$ = extension ratio reckoned on extended length,

$E_2 = \dfrac{l_1 - l_0}{l_0} = \dfrac{E_1 l_1}{l_0} =$ „ „ original length,

B = force applied to produce breakage divided by original area of cross-section.

P = intrinsic strength of the material.

Taking first the extension corresponding to (1), since, as is well known, the density of the material is not appreciably affected by the extension,

$$l_0 a_0^2 = l_1 a_1^2, \quad \text{so that} \quad E_1 = (a_0^2 - a_1^2)/a_0^2.$$

Now

$$B = Pa_1^2/a_0^2; \quad \text{thus} \quad B + 100E = Pa_1^2/a_0^2 + 100 - 100a_1^2/a_0^2. \qquad \text{(I)}$$

If the unit of force in which P is expressed is chosen so that P = 100 units and 100 units = 68 tons per square inch, $B + 100E_1 = 100$ units identically, if P is a constant. When E_1 is small, E_1 and E_2 are nearly equal, so that for small extensions

$$B + 100E_2 = 100 \text{ units nearly.}$$

For the extension corresponding to (2) take the axis of X parallel to the axis of the test-piece (4), and let x_2, x_3, be the abscissæ respectively of the broken end of the test-piece and the end (or mark) from which the extension is measured, and let y be the diameter at x.

The curve representing the diameter of the test-piece will be part of the curve $y = a(1 - e^{-x/c})$, and the condition as to the constant density (*i.e.*, invariable volume of the material) gives the relation

$$\int_{x_2}^{x_3} y^2 dx = a_0^2 l_0.$$

Performing the integration, we have

$$\frac{x_3-x_2}{l_0} = 1+\frac{c}{l_0}\,\frac{y_3-y_2}{y_3}+\frac{c}{2l_0}\,\frac{y_3^2-y_2^2}{y_3^2}, \qquad \text{(II)}$$

$$E_2 = \frac{c}{l_0}\,\frac{y_3-y_2}{y_3}+\frac{c}{2l_0}\,\frac{y_3^2-y_2^2}{y_3^2}.$$

We may take y_3 as nearly equal to a_1, and if, for convenience, we put c equal to a_1, c being the value of x for which $(a_1-y)/a_1 = e$, then

$$B+100E_2 = Py_2^2/a_0^2+100\{(a_1-y_2)/l_0+(a_1^2-y_2^2)/2l_0a_1\}. \qquad \text{(III)}$$

To find the actual value of E by computation is laborious, but a simple graphical method can be employed (fig. 3).

We have on putting $l_0 = 1$, and $c = a_1$ in (II),

$$x_3-y_3-\tfrac{1}{2}y_3^2-1 = x_2-y_2-\tfrac{1}{2}y_2^2.$$

Drawing a curve for $y+\frac{1}{2}y^2 = z$, say, x_2 can be found for any assumed value of x_3 thus:—From x_3 subtract z_3+1, and through the point $x_3-(z_3+1)$ on the axis of x, and at an angle of 45° to the axis, draw a straight line to cut curve z in z_2, then the abscissa of z_2 is x_2.

In this way the diagram (fig. 4) has been constructed for test-pieces of the proportions which were used for measurement, viz. ($a = 0{\cdot}564$ inch, $2l_0 = 2$ inches).

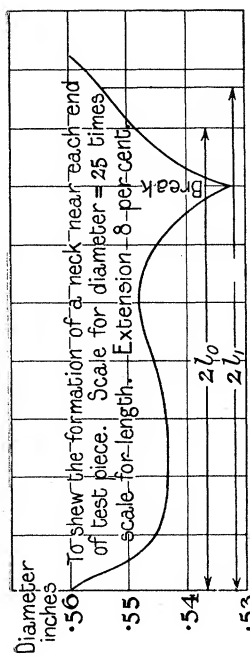
An examination of this diagram shows how nearly constant the sum of the breaking stress and elongation per cent. is on the assumption that the intrinsic strength of the material is a quantity which is not altered by heat treatment, whether of hardening or annealing. It gives some evidence, however, that the intrinsic strength increases slightly as the material is extended, though not more than about 5 per cent. for an extension of 30 per cent.

The intrinsic strength of steel probably varies with the amount of carbon, but since Professor Ewing gives 60 to 70 tons per square inch as the average tensile strength of high carbon tempered steel, in which the extension must be small, the variation cannot be large.

Any considerable addition of nickel or chrome, however, seems to increase the intrinsic strength, as the same authority gives 90 tons as the breaking stress of a 12-per-cent. nickel steel and 80 tons for a chrome steel (percentage of chrome not stated).*

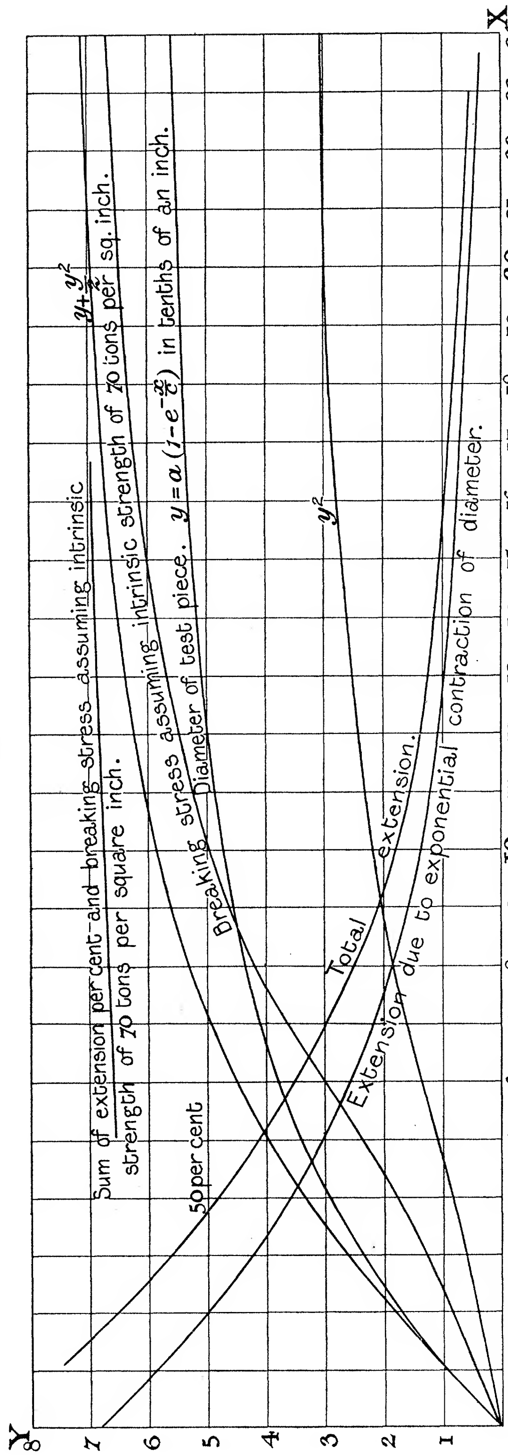
In saying that the intrinsic strength of all ordinary steels is about 70 tons per square inch, I exclude altogether the case of drawn wire or rolled plates, and refer only to such steel as is fairly isotropic and homogeneous at the

* "Strength of Materials," Ewing, 1906.



Diameter inches
·56
·55
·54
·53
To shew the formation of a neck near each end of test piece. Scale for diameter = 25 times scale for length. Extension 8 per cent.
Break
$2l_0$
$2l_1$

Fig. 5.



Y
X
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
1 2 3 4 5 6 7 8
Sum of extension per cent and breaking stress assuming intrinsic strength of 70 tons per square inch.
$y+\frac{y^2}{2}$
Breaking stress assuming intrinsic strength of 70 tons per sq. inch.
Diameter of test piece. $y=a(1-e^{-\frac{x}{c}})$ in tenths of an inch.
50 per cent
y^2
Total extension.
Extension due to exponential contraction of diameter.

Fig. 4.

commencement of the test. To compare the strength of a wire with the strength of the isotropic material is rather analogous to comparing the strength of a rope with strength of a felt made of the same fibre. The resistance of the rope to tension would be somewhat less than three times the resistance of the felt. The tensile strength of steel wire has been raised in some cases to more than twice the intrinsic strength of the material, but it is probable that, if the experiment could be tried, it would be found that the resistance to tension along a diameter of the wire was diminished.

When the condition regarding homogeneity and freedom from initial strain is fulfilled, I think the constancy of the sum of the breaking stress and extension per cent., and the approximation of this sum to 68, may be looked on as a good test of the quality and soundness of the steel.

If the sum is less than 68, it is an indication either of flaws or irregularities of structure, for it may be seen that the mere presence of parts having a different degree of ductility, without any actual flaws, would lessen the extension, because the less ductile parts would either give way first and so throw excessive stress on the rest, or if the harder parts did not extend with the rest the distortion of the neighbouring softer part would be excessive, and so cause a breakage.

It must be noted, however, that unsymmetrical holding of the test-piece in the testing machine would produce the same result, especially in the case of hard steels where the extension is small. See Nos. 14 to 19 and 21 to 23 in the Table, where the small extension is probably due, in part at any rate, to this cause.

It occasionally happens that the sum is greater than 68. This, I think, is due to the fact that what is called the breaking stress is the greatest stress measured, which is not always the stress at the moment of breakage. With very extensible steels the material at the neck gives way at the surface, while still holding on in the interior. Thus the working area at the moment of the break is somewhat less than the area of the broken surface, and the registered breaking stress in such cases refers to a period of the test before the extension has reached its ultimate value.

As far as I have observed the test-pieces rarely break in the middle of their length, and there is a tendency for a neck to be formed near both ends. Diagram 5 gives the results of some measures of diameter which show this. It is probably due to the sudden alteration in the diameter of the test-piece beyond the working part, which causes a non-uniform distribution of stress over the cross-sections in the neighbourhood. It should not appear in a test-piece where the diameter tapered gently to the working diameter.

A Table is appended giving the numerical results of some of the observations.

The general conclusion which I draw from the relations observed to hold between breaking stress extension and contraction of area, is that the various treatments and chemical compositions of ordinary mild steels (whether carbon nickel or nickel chrome), though operating powerfully on the limits of distortion, have but little effect on the intrinsic strength of the material.

Test-piece.	E.	B.	S.	D.	F.	Remarks.	
1	30 ·5	39 ·0	69 ·5	0 ·419	71	F.	F. = fibrous fracture.
2	29 ·0	37 ·5	66 ·5	0 ·412	70	F.	L. = laminated ,,
3	29 ·0	41 ·0	70 ·0	0 ·438	68	N. F.	G. = granular ,,
4	27 ·5	41 ·0	68 ·5	0 ·416	75	F.	
5	26 ·0	42 ·5	68 ·5	0 ·432	72	L.	
6	24 ·5	42 ·5	67 ·0	0 ·440	70	N. L.	
7	24 ·0	42 ·5	66 ·5	0 ·450	67	L.	
8	23 ·0	46 ·0	69 ·0	0 ·444	74	N. L.	
9	22 ·6	46 ·0	68 ·6	0 ·430	78	F.	
10	22 ·5	46 ·0	68 ·5	0 ·435	77	F.	
11	21 ·0	45 ·0	66 ·0	0 ·440	75	L.	
12	20 ·0	47 ·0	67 ·0	0 ·450	73	L.F.	
13	20 ·0	47 ·0	67 ·0	0 ·440	77	L.F.	
14	16 ·0	43 ·5	59 ·5	0 ·483	59	U.G.	E. too small, B. too small.
15	15 ·0	43 ·5	58 ·5	0 ·496	56	U.G.	E. ,, B. ,,
16	15 ·0	44 ·0	59 ·0	0 ·480	61	U.G. (slightly),	E. ,, B. ,,
17	15 ·0	43 ·5	58 ·5	0 ·444	70	N.U.L. ,,	E. ,, B. correct probably.
18	13 ·0	52 ·0	65 ·0	0 ·520	61	N.U.G.	E. ,, B. too small.
19	13 ·0	52 ·0	65 ·0	0 ·523	60	N.U.G.	E. ,, B. ,,
20	12 ·5	46 ·5	59 ·0	0 ·451	72	L.	
21	10 ·0	50 ·3	60 ·3	0 ·534	56	U.G. (slightly),	E. ,, B. ,,
22	9 ·0	48 ·0	57 ·0	0 ·530	65	U.L. ,,	E. ,, B. ,,
23	8 ·0	56 ·0	64 ·0	0 ·527	64	U.G. ,,	E. ,, B. ,,

E. = Extension per cent. on a length of 2 inches.
B. = Breaking stress in tons per square inch of original cross-section.
S. = Sum of breaking stress and extension.
D. = Measured diameter at break.
F. = Intensity of stress at broken surface (to the nearest ton per square inch) on the assumption that B. was the stress at the time breakage occurred.

N denotes nickel steel. U. means that the axis of the test-piece was bent, as well as extended. This may be due to the want of homogeneity, but is more probably the result of the test-piece not being held symmetrically in the testing machine.